CONIC SECTIONS

General comments

The conic sections are two-dimensional (flat) figures. Therefore, assume that all descriptions are for a plane.

In each of the following, the figure is in standard position. To obtain the general equation, replace \( x \) by \( x-h \) and replace \( y \) by \( y-k \).

Circle

Definition: A circle consists of all points that are a given distance from some fixed point.

![Circle Diagram]

Equation: \( x^2 + y^2 = r^2 \)

Note: A circle is a special case of an ellipse where \( a = b = r \).

- center at \((0,0)\)
- \( r \) = radius
- eccentricity = \( e = 0 \)

Parabola

Definition: A parabola is the set of all points that are equidistant from a fixed line (directrix) and a fixed point (focus) not on the line.

![Parabola Diagram]

Equations: \( x^2 = 4py \) (opens up) \( y^2 = 4px \) (opens right)
\( x^2 = -4py \) (opens down) \( y^2 = -4px \) (opens left)

- vertex at \((0,0)\)
- \( p \) = distance between focus and vertex = distance between vertex and directrix
**Ellipse**

*Definition:* An ellipse is the set of all points the sum of whose distances from two distinct fixed points (foci) is a constant.

![Ellipse Diagram]

*Equations:*  
\[ \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \]

- Center at (0,0)
- \(a = 1/2\) major axis
- \(b = 1/2\) minor axis
- \(c = \text{distance from center to focus}\)

- Eccentricity: \(e = \frac{c}{a}\) and \(0 < e < 1\)

**Hyperbola**

*Definition:* A hyperbola is the set of all points the difference of whose distances from two distinct fixed points (foci) is constant.

![Hyperbola Diagram]

*Equations:*  
\[ \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \]

- Center at (0,0)
- \(c = \text{distance between center and focus}\)
- \(a = \text{distance between center and vertex}\)
- Transverse axis = \(2a\)

- Eccentricity: \(e = \frac{c}{a}\) and \(e > 1\)

**Classifying a conic from its general equation**

The graph of \(Ax^2 + Cy^2 + Dx + Ey + F = 0\) is one of the following (except in degenerate cases).

- If \(A = C\), then it is a circle. (Note that a circle can be considered as a special ellipse.)
- If \(A = 0\) or \(C = 0\) (but not both), then it is a parabola.
- If \(A\) and \(C\) have like signs, then it is an ellipse.
- If \(A\) and \(C\) have opposite signs, then it is a hyperbola.