Factoring

Worked Examples
Factor completely: \[ y^5 - 6y^4 + 9y^3 \]
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First, factor out the common factors:

\[ y^5 - 6y^4 + 9y^3 = y^3(y^2 - 6y + 9) \]
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Now you should recognize what remains as a perfect square:

\[
y^5 - 6y^4 + 9y^3 = y^3(y^2 - 6y + 9) = y^3(y - 3)^2
\]
Factor completely: $8x^2 - 11x - 7 \frac{1}{2}$
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\[
8x^2 - 11x - 7 \frac{1}{2} = 8x^2 - 15x + 4x - 7 \frac{1}{2} = \\
x(8x - 15) + \frac{1}{2}(8x - 15) = \left( x + \frac{1}{2} \right)(8x - 15)
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\[x(8x - 15) + \frac{1}{2}(8x - 15) = \left(x + \frac{1}{2}\right)(8x - 15)\]

But what if you can’t find those two numbers? Or what if the factorization by grouping is too tricky?
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But what if you can’t find those two numbers? Or what if the factorization by grouping is too tricky? You can use the quadratic formula to find the same factors:

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(8)(-7.5)}}{2(8)} = \frac{11 \pm \sqrt{361}}{16} = \frac{11 \pm 19}{16}.
\]

The two roots are \(x = \frac{15}{8}\) and \(x = -\frac{1}{2}\). So this quadratic has \(x - \frac{15}{8}\) and \(x + \frac{1}{2}\) as factors. Multiplying by 8 so the leading coefficient is right, we get:

\[
8x^2 - 11x - 7 \frac{1}{2} = 8 \left( x - \frac{15}{8} \right) \left( x + \frac{1}{2} \right).
\]

(You should confirm that this is the same answer that we got from the AC method.)
Factor completely: $y^3 - 11y^2x + 24yx^2$
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First, factor out the common factor of \( y \):

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y^3 - 11y^2 x + 24yx^2 = y(y^2 - 11yx + 24x^2).
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Now don’t panic because there are still two variables – the remaining piece has that same quadratic pattern.
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First, factor out the common factor of \( y \):

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y^3 - 11y^2x + 24yx^2 = y(y^2 - 11yx + 24x^2).
\]

Now don’t panic just because there are still two variables – the remaining piece has that familiar quadratic pattern.

We can factor by guess and check. We need two numbers that add to \(-11\) and multiply to make \(24\). \(-8\) and \(-3\) work. So

\[
y^3 - 11y^2x + 24yx^2 = y(y^2 - 11yx + 24x^2) = y(y - 3x)(y - 8x).
\]
Factor completely: \[ a^4 - 5a^2 - 36 \]
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Although this isn’t a quadratic, it has that pattern – if you think of \( a^2 \) as your variable. The “quadratic” factors by guess and check (two numbers that add to –5 and multiply to make –36):

\[ a^4 - 5a^2 - 36 = (a^2 - 9)(a^2 + 4) \]
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$$a^4 - 5a^2 - 36 = (a^2 - 9)(a^2 + 4)$$

Now the factors really are quadratics that we recognize. The first is a difference of squares, and the second is a sum of squares (which is already completely factored).

$$a^4 - 5a^2 - 36 = (a^2 - 9)(a^2 + 4) = (a + 3)(a - 3)(a^2 + 4)$$